OCR
RECOGNISING ACHIEVEMENT

Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS

4727 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The plane p has equation $\mathbf{r}.(\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 4$ and the line l_1 has equation $\mathbf{r} = 2\mathbf{j} - \mathbf{k} + t(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The line l_2 is parallel to p and perpendicular to l_1, and passes through the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Find the equation of l_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[4]**

2 **(i)** Solve the equation $z^4 = 2(1 + \mathrm{i}\sqrt{3})$, giving the roots exactly in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[5]**

(ii) Sketch an Argand diagram to show the lines from the origin to the point representing $2(1 + \mathrm{i}\sqrt{3})$ and from the origin to the points which represent the roots of the equation in part **(i)**. **[3]**

3 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\cot x = 2x$$

for which $y = 2$ when $x = \frac{1}{6}\pi$. Give your answer in the form $y = \mathrm{f}(x)$. **[9]**

4 The elements a, b, c, d are combined according to the operation table below, to form a group G of order 4.

	a	b	c	d
a	b	a	d	c
b	a	b	c	d
c	d	c	a	b
d	c	d	b	a

Group G is isomorphic **either** to the multiplicative group $H = \{e, r, r^2, r^3\}$ **or** to the multiplicative group $K = \{e, p, q, pq\}$. It is given that $r^4 = e$ in group H and that $p^2 = q^2 = e$ in group K, where e denotes the identity in each group.

(i) Write down the operation tables for H and K. **[4]**

(ii) State the identity element of G. **[1]**

(iii) Demonstrate the isomorphism between G and either H or K by listing how the elements of G correspond to the elements of the other group. If the correspondence can be shown in more than one way, list the alternative correspondence(s). **[4]**

5 **(i)** By expressing $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, prove that

$$\sin^3\theta\cos^2\theta \equiv -\tfrac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta).$$ **[6]**

(ii) Hence show that all the roots of the equation

$$\sin 5\theta = \sin 3\theta + 2\sin\theta$$

are of the form $\theta = \dfrac{n\pi}{k}$, where n is any integer and k is to be determined. **[3]**

6 The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 12e^{2x}.$$

(i) Find the general solution of the differential equation. **[6]**

(ii) It is given that the curve which represents a particular solution of the differential equation has gradient 6 when $x = 0$, and approximates to $y = e^{2x}$ when x is large and positive. Find the equation of the curve. **[4]**

7 With respect to the origin O, the position vectors of the points U, V and W are $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively. The mid-points of the sides VW, WU and UV of the triangle UVW are M, N and P respectively.

(i) Show that $\overrightarrow{UM} = \frac{1}{2}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$. **[2]**

(ii) Verify that the point G with position vector $\frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$ lies on UM, and deduce that the lines UM, VN and WP intersect at G. **[5]**

(iii) Write down, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line through G which is perpendicular to the plane UVW. (It is not necessary to simplify the expression for $\mathbf{b}$.) **[2]**

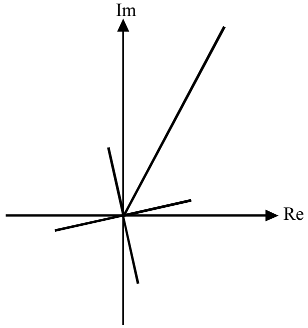
(iv) It is now given that $\mathbf{u} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{v} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix}0\\0\\1\end{pmatrix}$. Find the perpendicular distance from O to the plane UVW. **[3]**

8 The set M of matrices $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, where a, b, c and d are real and $ad - bc = 1$, forms a group $(M, \times)$ under matrix multiplication. R denotes the set of all matrices $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$.

(i) Prove that $(R, \times)$ is a subgroup of $(M, \times)$. **[6]**

(ii) By considering geometrical transformations in the x-y plane, find a subgroup of $(R, \times)$ of order 6. Give the elements of this subgroup in exact numerical form. **[5]**

Question			Answer	Marks	Guidance	
1			METHOD 1			
			$\mathbf{b} = [1, -3, 4] \times [3, 1, 2] = [-10, 10, 10]$	M1	For attempt to find vector product of directions	
			$= k[-1, 1, 1]$	M1	Correct calculation of vector product	Allow 1 error
				A1	For correct **b** .	
			$\Rightarrow \ \mathbf{r} = [1, 4, 2] + t[-1, 1, 1]$	B1 FT	For correct equation. FT from **b**	
				[4]		
			METHOD 2			
			$[x, y, z] \cdot [1, -3, 4] = 0 \ \Rightarrow \ x - 3y + 4z = 0$		For an equation from l_2 perpendicular to normal of plane	
			$[x, y, z] \cdot [3, 1, 2] = 0 \ \Rightarrow \ 3x + y + 2z = 0$	M1	and an equation from l_2 perpendicular to l_1	
			Solving $\Rightarrow [x, y, z] = \mathbf{b} = k[-1, 1, 1]$	M1 A1		
			$\Rightarrow \ \mathbf{r} = [1, 4, 2] + t[-1, 1, 1]$	B1FT	For correct equation. FT. from **b**	Must show "**r** ="
2	(i)		$z^4 = 4\left(\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}\right) = 4\,\mathrm{cis}\,\frac{1}{3}\pi$	B1	For $\arg(z^4) = \frac{1}{3}\pi$ soi	
				M1	For dividing $\arg(z^4)$ by 4	
			$z = \sqrt{2}\,\mathrm{cis}\left(k\frac{\pi}{12}\right), \quad k = 1, 7, 13, 19$	A1	For any 2 correct values of k	
				A1	For all 4 values of k *and no extras.* Ignore values outside range	For second A1, must be in correct form.
				B1	For modulus of all stated roots = $\sqrt{2}$	Don't accept 1.41.. or $\sqrt[4]{4}$
					SR For $\arg(z^4) = \frac{1}{6}\pi$ award B0 M1 A1 FT for all $\mathrm{cis}\left(k\frac{\pi}{24}\right), k = 1, 13, 25, 37$, A0 B0/B1	
				[5]		

Question			Answer	Marks	Guidance	
2	(ii)			B1	For roots forming a square, centre O, on equal-scale axes.	Must be roots distinct from z^4 Penalise once use of points not lines
				B1	For z^4 and only one root in first quadrant with arguments in ratio approximately 3:1	
				B1	For $\lvert z^4\rvert : \lvert z\rvert \approx 4:\sqrt{2}$ (allow (2,4):1)	For all four roots
				[3]		
3			Integrating factor $= e^{\int \cot x\, dx} = e^{\ln \sin x} = \sin x$	M1	For IF $= e^{\pm \ln \sin x}$ *OR* $e^{\pm \ln \cos x}$	
				A1	For simplified IF	
			$\Rightarrow \frac{d}{dx}(y \sin x) = 2x \sin x$	M1	For $\frac{d}{dx}(y.\text{their IF}) = 2x.\text{their IF}$	
			$\Rightarrow y \sin x = -2x \cos x + \int 2\cos x\, dx$	M1*	For attempt to integrate RHS using parts for $\int x \begin{cases}\sin x \\ \cos x\end{cases} dx$	(Must use u = (2)x)
				A1	For correct RHS 1st stage	
			$\Rightarrow y \sin x = -2x \cos x + 2\sin x\ (+c)$	A1	oe	
			$\left(\frac{1}{6}\pi, 2\right) \Rightarrow c = \frac{1}{6}\pi\sqrt{3}$	M1dep *	For substituting $\left(\frac{1}{6}\pi, 2\right)$ into their GS (with c)	c = 0.907
				A1 FT	For correctly finding c (FT from GS)	
			$\Rightarrow y = -2x \cot x + 2 + \frac{1}{6}\pi\sqrt{3}\operatorname{cosec} x$	A1	For correct solution **AEF** of standard notation $y = f(x)$	
				[9]		

Question			Answer	Marks	Guidance	
4	(i)		$\begin{array}{c\|cccc} H & e & r & r^2 & r^3 \\ \hline e & e & r & r^2 & r^3 \\ r & r & r^2 & r^3 & e \\ r^2 & r^2 & r^3 & e & r \\ r^3 & r^3 & e & r & r^2 \end{array}$ $\begin{array}{c\|cccc} K & e & p & q & pq \\ \hline e & e & p & q & pq \\ p & p & e & pq & q \\ q & q & pq & e & p \\ pq & pq & q & p & e \end{array}$	B2 B2 **[4]**	For correct table for H For correct table for K **SR** In both tables allow B1 for 1 or 2 errors	
4	(ii)		Identity = b	B1 **[1]**	For correct identity	
4	(iii)		G is isomorphic to H $\begin{array}{c\|c\|c} G & H & H \\ \hline a & r^2 & r^2 \\ b & e & e \\ c & r & r^3 \\ d & r^3 & r \end{array}$	B1 B1 B1 B1 **[4]**	For H identified as isomorphic to G (may be implied by table) For $a \leftrightarrow r^2$ at least once For $c, d \leftrightarrow r, r^3$ either way For $c, d \leftrightarrow r, r^3$ both ways **and** b corresponds to e explicit. Award fourth **B1** only for completely correct answer. If none of last 3 marks gained, then **SC1** for order of all elements of G and H	
5	(i)		METHOD 1 $\sin^3\theta\cos^2\theta = \left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^3\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^2$ $= -\frac{1}{32i}\left(z^3-3z+3z^{-1}-z^{-3}\right)\left(z^2+2+z^{-2}\right)$ $= -\frac{1}{32i}\left(\left(z^5-z^{-5}\right)-\left(z^3-z^{-3}\right)-2\left(z-z^{-1}\right)\right)$ $= -\frac{1}{16}\left(\frac{z^5-z^{-5}}{2i}-\frac{z^3-z^{-3}}{2i}-2\frac{z-z^{-1}}{2i}\right)$ $= -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta)$	 B1 M1 M1 B1 M1 A1 **[6]**	z may be used for $e^{i\theta}$ throughout For $\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)$ *OR* $\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)$ soi For expanding brackets (binomial theorem or otherwise) For full expansion with 12 terms. For $-\frac{1}{32i}$ For grouping terms This step, oe, is needed for the final mark For simplification to **AG** WWW	 two brackets expanded soi by alternate method Can be seen at any stage oe includes replacing z^5-z^{-5} with 2isin5θ etc

Question			Answer	Marks	Guidance	
			METHOD 2 $\sin^3\theta\cos^2\theta = \sin^3\theta - \sin^5\theta$ $2i\sin\theta = z - \frac{1}{z}$ $-8i\sin^3\theta = z^3 - 3z + \frac{3}{z} - \frac{1}{z^3}$ $= (z^3 - \frac{1}{z^3}) - (3z - \frac{3}{z})$ $= 2i\sin 3\theta - 6i\sin\theta$ $32i\sin^5\theta = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ $= (z^5 - \frac{1}{z^5}) - (5z^3 - \frac{5}{z^3}) + (10z - \frac{10}{z})$ $= 2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta$ $\sin^3\theta\cos^2\theta$ $= -\frac{1}{32i}(4(2is3\theta - 6is\theta) + (2is5\theta - 10is3\theta + 20is\theta))$ $= -\frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 4\sin 3\theta + 10\sin\theta - 12\sin\theta)$ $= -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta)$	B1 M1 M1 B1 B1 A1	 For RHS * For grouping terms For RHS of this line and line * above For $-\frac{1}{32i}$ For **ag** www	
5	(ii)		$\sin^3\theta\cos^2\theta = 0 \Rightarrow \sin\theta = 0$ *OR* $\cos\theta = 0$ $\Rightarrow \theta = r\pi$ *OR* $\theta = (2r+1)\frac{1}{2}\pi$ $\Rightarrow \theta = \frac{n\pi}{2}$	M1 A1 A1 **[3]**	For either equation Accept also Sinθ = +/-1 For either solution, **AEF** including a list of the first few For both of above solutions leading to general solution in form of **AG** where $k = 2$	Can be implied by the A mark plus at least $\sin^3\theta = 0$ or similar. At least 2 in list (and no wrong solution)

Question			Answer	Marks	Guidance	
6	(i)		METHOD 1			
			$m^2+4m=0 \Rightarrow m=0,-4$	M1	For attempt to solve correct auxiliary equation	
			CF = $A+Be^{-4x}$	A1	For correct CF	
			PI $y=pe^{2x} \Rightarrow 4p+8p=12$	B1	For PI of correct form seen	Beware poor use of pxe^{2x}
				M1	For differentiating PI and substituting	Scores maximum
			$\Rightarrow p=1$	A1	For correct p	of **M1 A1 B0 M1**
			GS $y=A+Be^{-4x}+e^{2x}$	B1 FT	For using GS = CF + PI with 2 arbitrary constants in GS and none in PI	**A0 B0**
				[6]		
			METHOD 2			
			Integrating $\Rightarrow \frac{dy}{dx}+4y=6e^{2x}+c$	M1 B1	For attempt to integrate equation For $+c$ included	
			IF $e^{4x} \Rightarrow \frac{d}{dx}\left(ye^{4x}\right)=6e^{6x}+ce^{4x}$	B1√ M1	For correct IF. f.t. from their DE For multiplying through by their IF and attempting to integrate	
			$\Rightarrow ye^{4x}=e^{6x}+\frac{1}{4}ce^{4x}+B$	A1	For correct integration both sides, including $+B$	
			$\Rightarrow y=e^{2x}+A+Be^{-4x}$	A1	For correct solution	Must include "y ="
6	(ii)		$\frac{dy}{dx}=-4Be^{-4x}+2e^{2x}$	M1	For differentiating "their GS" with 2 arbitrary constants and substituting values to obtain an equation	If "their CF" is $(A+Bx)e^{-4x}$
			$\left(0,\frac{dy}{dx}=6\right) \Rightarrow -4B+2=6 \Rightarrow B=-1$	A1	For correct B	can score max of **M1 A0 B1 A0**
			$(y\approx e^{2x} \Rightarrow) A=0$	B1	For correct A and consistent with" their GS"	
			$\Rightarrow y=-e^{-4x}+e^{2x}$	A1	For correct equation **www**	
				[4]		
7	(i)		$\mathbf{m}=\mathbf{v}+\frac{1}{2}(\mathbf{w}-\mathbf{v}) \Rightarrow$	M1	For using vector triangle, or equivalent, for M	$\overrightarrow{UM}=\overrightarrow{UV}+\overrightarrow{VM}$ $=(\mathbf{v}-\mathbf{u})+\frac{1}{2}(\mathbf{w}-\mathbf{v})$
			$\overrightarrow{UM}=\mathbf{v}+\frac{1}{2}(\mathbf{w}-\mathbf{v})-\mathbf{u}=\frac{1}{2}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$	A1	For correct expression **AG**	
					SR Allow use of ratio theorem	Minimum $-\mathbf{u}+\frac{1}{2}(\mathbf{v}+\mathbf{w})$
				[2]		

Question			Answer	Marks	Guidance	
7	(ii)		METHOD 1 (first 3 marks) $\overrightarrow{UM}$ is $\mathbf{r} = \mathbf{u} + \frac{1}{2}t(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ $t = \frac{2}{3} \Rightarrow \mathbf{u} + \frac{1}{3}(\mathbf{v} + \mathbf{w} - 2\mathbf{u}) = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$ METHOD 2 (first 3 marks) $\overrightarrow{UG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) - \mathbf{u} = \frac{1}{3}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ OR $\overrightarrow{MG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) - \frac{1}{2}(\mathbf{v} + \mathbf{w}) = -\frac{1}{6}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ $\Rightarrow U, G, M$ collinear By symmetry of $\overrightarrow{OG}$ in **u**, **v**, **w** G also lies on VN, WP $\Rightarrow UM, VN, WP$ intersect at G	M1* M1* A1 M1* M1* A1 B1 B1dep * **[5]**	For equation of *UM* For attempt to find a suitable value of *t* For $t = \frac{2}{3}$ and *G* obtained **AG** For finding directions of *UG* or *MG* For comparing with *UM* For showing *G* lies on *UM* **AG** For use of symmetry, or by repeating method for UM twice more. For complete reasoning to **AG**	
7	(iii)		Line is $\mathbf{r} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) + t(\mathbf{u} - \mathbf{v}) \times (\mathbf{u} - \mathbf{w})$ (etc)	B1 B1 **[2]**	For $r = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) + t \times$ "any vector" For a correct **n**, using any 2 of $\pm(\mathbf{u} - \mathbf{v})$, $\pm(\mathbf{v} - \mathbf{w})$, $\pm(\mathbf{w} - \mathbf{u})$	Allow $\overrightarrow{UV} \times \overrightarrow{VW}$ or similar

Question			Answer	Marks	Guidance	
7	(iv)		METHOD 1 $\mathbf{n} = [1, 0, -1] \times [0, 1, -1]$ (etc) $= k[1, 1, 1]$	M1*	For attempt to find **n**	May see use of $\frac{\lvert p.n - d \rvert}{\lvert n \rvert}$
			UVW is $\mathbf{r}.\mathbf{n} = [1, 0, 0].[1, 1, 1] = 1$	M1dep*	For substituting a point	
			$\Rightarrow d = \frac{1}{\sqrt{3}}$	A1	For correct d	
				[3]		
			METHOD 2 UVW is $x + y + z = 1$ (from given **u**, **v**, **w**)	M2	For attempt to find cartesian equation	
			$\Rightarrow d = \frac{1}{\sqrt{3}}$	A1	For correct d	
			METHOD 3 $\overrightarrow{OG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$	M1*	For stating or implying $\lvert\overrightarrow{OG}\rvert$ is d	
			$\Rightarrow OG = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}}$	M1dep*	For finding magnitude	
			$\Rightarrow d = \frac{1}{\sqrt{3}}$	A1	For correct d	

Question			Answer	Marks	Guidance	
8	(i)		For R, $\cos^2\theta+\sin^2\theta=1$ ⇒ad-bc =1 (⇒ $R\subset M$)	B1	For showing $R\subset M$	
			$R(\theta)R(\phi)=R(\theta+\phi)$ and hence closed, since	M1	For multiplying 2 distinct elements	
			$\cos\theta\cos\phi-\sin\theta\sin\phi=\cos(\theta+\phi)$ and			
			$\pm(\cos\theta\sin\phi+\sin\theta\cos\phi)=\pm\sin(\theta+\phi)$	A1	For obtaining $R(\theta)R(\phi)\in R$	Must demonstrate use of compound angles or explain rotations.
			Identity $\theta=0\Rightarrow\begin{pmatrix}1&0\\0&1\end{pmatrix}\in R$	B1	For identity element related to θ = 0	
			Inverse $R(-\theta)=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$	B1	For inverse element …	
			$=\begin{pmatrix}\cos(-\theta)&-\sin(-\theta)\\\sin(-\theta)&\cos(-\theta)\end{pmatrix}$	B1	...converted to form of elements of R	
				[6]		
			SR For use of $\left(a,b\in R\Rightarrow ab^{-1}\in R\right)\Leftrightarrow R$ is a subgroup of M			
			For R, $\cos^2\theta+\sin^2\theta=1\Rightarrow R\subset M$	B1	For showing $R\subset M$	
			$R(\theta)R(\phi)^{-1}=\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}\cos(-\phi)&-\sin(-\phi)\\\sin(-\phi)&\cos(-\phi)\end{pmatrix}$	B1 B1 M1 A1	For considering $R(\theta)R(\phi)^{-1}$ For correct inverse For multiplying elements For correct product	
			$=\begin{pmatrix}\cos(\theta-\phi)&-\sin(\theta-\phi)\\\sin(\theta-\phi)&\cos(\theta-\phi)\end{pmatrix}\in R$			
			Set is non-empty	B1	Can be implied by identity element related to θ = 0	

Question			Answer	Marks	Guidance	
8	(ii)		For $\theta = \frac{1}{3}k\pi$ elements are	B1	For $\theta = \frac{1}{3}\pi$ soi	Allow degrees instead of radians.
			$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}$,	M1	For using "their θ" in $\begin{pmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{pmatrix}$ for at least 2 values of k, or lists all 6 values of θ	
				A1	For identity and one other element other than (-I)	
			$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$	A1 A1 **[5]**	For 2 more elements For all 6 elements correct	